Node Centrality Measures are a poor substitute for Causal Inference

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Background

Theoretical network models:
- Constitute a shift in how we view psychological constructs.
- Example: Depression does not cause symptoms such as worry, sadness, and lack of energy but rather arises out of their direct causal interactions [1].

Statistical network models:
- Majority of research uses Gaussian graphical model and cross-sectional data [2].
- Node centrality measures used to gauge importance of nodes; many researchers suggest that highly central nodes may constitute good intervention targets [2].

Observations:
- Mismatch between theoretical and statistical models.
- Intervention is a causal notion. Here [3] we ask: how well do node centrality measures map onto measures of causal influence?

Node-specific Measures

Node Centrality Measures

There exist a number of node centrality measures for weighted graphs [5]. We focus on degree, strength, closeness, betweenness, and eigenvector-centrality.

Causal Influence Measures

We use the following two causal influence measures [4]:

\[ ACE(X_j \rightarrow X_i) := \mathbb{E}[X_i|do(X_j = x_j)] - \mathbb{E}[X_i|do(X_j = x_j + 1)] \]  
\[ CE_{bc}(X_j \rightarrow X_i) := KL(P_i|P_j) = \frac{1}{2} \left( \log \frac{\det \Sigma_j^{1/2}}{\det \Sigma_j} - \log \frac{\det \Sigma_i^{1/2}}{\det \Sigma_i} - n \right), \]

where we compute the total causal influence of node \( X_j \) by summing its causal influence on all of its children.

Simulation setup

For each network type (Figure 1), we randomly generate a DAG with number of nodes \( n = \{10, 20, 30, 40, 50\} \) and network density \( d = \{0.1, 0.2, \ldots, 0.9\} \). We sample \( n = 5000 \) observations, estimate a Gaussian graphical model, and compute (a) the rank-correlation between node centrality and causal influence and (b) the probability of correctly classifying the node with the highest causal influence based on centrality.

Example

A True DAG Estimated MRF B Distribution of node scores

Figure 2: A shows the true DAG and the estimated undirected network for \( n = 10 \) and \( d = 0.3 \). B shows the distributions of \( CE_{bc} \) and betweenness centrality (\( r = 0.32 \)). Interactive plot: https://fdabl.shinyapps.io/causality-centrality-app/.

Results

Figure 3 shows a homogenous picture of correlations across graphs. The average rank-correlation is between 0.30 and 0.50, with 10% and 90% quantiles ranging from \(-0.10\) to \(0.70\). Eigenvector-centrality is the exception, because in contrast to the other measures, its distribution over nodes does not become left-tailed.

Figure 3: Shows rank-based correlation \( r \) of \( CE_{bc} \) with node centrality measures. Error bars denote 95% confidence intervals across the 500 simulations.

Figure 4: Shows probability of correctly classifying the node with highest \( CE_{bc} \) based on centrality measures. Error bars denote 95% confidence intervals across the 500 simulations.

Conclusion

- Overall weak relationship between node centrality and causal influence measures, except for eigenvector-centrality.
- If the underlying real-world system can be modeled as a DAG, but researchers interpret nodes with high centrality as causally important, then this may result in sub-optimal interventions.

References